J. Fluid Mech. (1982), *vol.* 118, *pp.* 41–58
Printed in Great Britain

The measurement of Reynolds stresses with a pulsed-wire anemometer

By I. P. CASTRO
Department of Mechanical Engineering, University of Surrey, Guildford, Surrey, U.K.
AND B. S. CHEUN
Department of Civil Engineering, University of Surrey, Guildford, Surrey, U.K.

(Received 13 March 1981 and in revised form 5 October 1981)

An investigation of the errors arising in pulsed-wire anemometer measurements of the Reynolds stresses in turbulent flows is described. Attention is concentrated first on a theoretical approach, in which an idealized yaw response and an assumed form for the joint velocity probability-density distribution are used to determine the errors in measurements of, principally, $\overline{uv}$ and $\overline{v^2}$ when the probe is used like an ordinary single slanted hot wire. Actual pulsed-wire measurements in a range of turbulent shear flows are then compared with crossed-hot-wire results and with the theoretically simulated pulsed-wire response obtained from calculations and the crossed-wire data. It is shown that whilst pulsed-wire measurements of lateral intensity and shear stress are inevitably rather unsatisfactory in regions of low intensity (less than 10 %, say) they agree reasonably well with crossed-wire measurements in flows where the intensities are higher, but do not exceed those for which sensible corrections to cross-wire data are possible (up to, say, 30 %). In this medium-intensity range, however, the pulsed-wire errors are found to be critically dependent on the finite limit of the pulsed wire's yaw response; it seems that acceptable measurements can only be made if this exceeds about 75°. Beyond an intensity of about 30 % the errors in $\overline{v^2}$ measurements (which are usually much higher than those for $\overline{uv}$) become less dependent on the exact nature of the yaw response and invariably decrease with increasing intensity. They can, with care, be made as low as 15 %. It is concluded that pulsed-wire measurements of the Reynolds stresses can be made with an accuracy similar to that of crossed-wire measurements in medium-intensity flows. Such measurements are certainly adequate for many practical purposes in high-intensity flows where hot-wire techniques are useless.

1. Introduction

Apart from laser–Doppler techniques, the pulsed-wire anemometer originally described by Bradbury & Castro (1971) is the only instrument capable in principle of making velocity and turbulence measurements in highly turbulent flows. It has been developed considerably over the years, so that, whilst early studies necessitated manual acquisition of the times of flight of the heat tracer (Bradbury 1969; Castro 1971), more recent work has used the instrument on-line to various desk-top calculators (Bradbury 1976; Castro & Robins 1977), minicomputers (Britter & Hunt 1979; Castro & Snyder

1981) or micro-computer-based systems (Eaton, Johnston & Jeans 1979). All this published work has thus far concentrated on measurements of longitudinal mean velocity, longitudinal turbulence intensity and probability-density distributions, but there is no reason, in principle, why the instrument should not be used to measure transverse mean and fluctuating velocities, simply by using it like an ordinary slant hot-wire anemometer. Indeed, Bradbury (1978) has made some preliminary measurements of transverse turbulent intensity and shear stress in that way. In addition to the usual uncertainties in pulsed-wire probe calibration (which need be no worse than hot-wire calibration errors) there are, of course, several sources of measurement error that arise when the probe is used in a turbulent flow. For example, in a shear flow at least one of the wires will be subject to a mean velocity gradient along its length, so that errors arising from wire-length effects might be significant if the scale of the mean flow is not large compared to the probe size. However, except at very high intensities (when other errors dominate) the wire *spacing* is the relevant length scale, and this is usually similar to a typical hot-wire length.

Now in the case of the single and crossed hot wire it is well known that the major sources of error, particularly in measurement of quantities involving the fluctuating transverse velocities, arise from uncertainties in the response of the wire to velocity components parallel to its axis. We believe that the largest errors in pulsed-wire measurements arise in an analogous way, and are a direct consequence of two major aspects of its yaw response, both of which have been outlined previously (Bradbury & Castro 1971; Bradbury 1976). Firstly, the probe geometry limits its yaw response (see figure 1) so that for instantaneous velocity vectors lying outside, say, a cone of semi-angle ϕ, where ϕ is typically about 70°, the instrument measures a zero velocity. Secondly, even for velocity vectors lying within that cone, thermal diffusion of the heat tracer prevents the probe from having a perfect cosine-law response. By assuming a representative form both for this imperfect response and for the joint probability-density distribution of the velocity it is possible to estimate the errors arising from the imperfect response. This has already been done for various, somewhat restricted cases (Bradbury & Castro 1971, Bradbury 1976), but no attempt has been made to generalize the calculations to take account of non-zero transverse mean velocities, non-zero shear stress, or non-identical values for the three components of turbulence intensity. In real measurement situations, particularly if the probe is rotated to obtain information concerning the transverse components, these three features of the flow 'seen' by the probe can obviously be important.

In this paper we investigate the more general case by, as before, assuming a Gaussian model for the joint probability-density distribution, but now without any restrictive assumptions concerning the various parameters that define it. The three-dimensional integrals that determine the probability of missing tracers (which, incidentally, can be useful in suggesting when errors in measurements using more standard instrumentation are likely to be significant) and the corresponding integrals that determine the errors in mean velocity and turbulent intensity all have to be evaluated numerically. This has been done for a variety of typical cases to demonstrate the effect of the imperfect yaw response on PWA measurements of the Reynolds stresses. These results are presented in § 2.

As a demonstration of the usefulness of the error estimates, § 3 contains a comparison between the results of PWA measurements of the Reynolds stresses in three flows covering a wide range of turbulence intensities, similar measurements obtained using standard crossed hot-wire techniques and the expected PWA results obtained by assuming that the hot-wire measurements are correct and deducing the PWA errors as in § 2. The comparisons are necessarily restricted to regions where the crossed hot wire can be fairly confidently used; elsewhere it would clearly be worth while to compare pulsed-wire measurements with those obtained using laser–Doppler techniques, but the present results are, nevertheless, useful and, taken with the results of § 2, are sufficient to give a feel for the likely magnitude of the errors in a wide range of turbulent intensities.

Section 4 summarizes the conclusions, and it is argued that, although the PWA is not a particularly accurate instrument when used to measure transverse intensities and shear stress, the errors are usually low enough, in regions where standard hot-wire techniques would be useless, to allow quite useful measurements to be made.

2. Theoretical results

If the PWA probe had an ideal cosine-law response the errors arising from the finite yaw response ($\phi < 90°$) might be quite small compared with those arising in hot-wire anemometry and, indeed, could possibly be made insignificant by improving the probe geometry to give ϕ values nearer to 90°. It would also be much more straightforward to calculate the errors arising from the finite yaw response than it is to calculate corresponding hot-wire errors, which do not arise simply from a sharp cut-off in the angular response. However, as indicated earlier, thermal diffusion degrades the PWA yaw response and for a total velocity $\tilde{U}$, inclined at θ to the direction normal to the three wires, it has been found that the total response can be reasonably approximated by

$$\tilde{U}_{\mathrm{m}} = \tilde{U}(\cos\theta + \epsilon\sin\theta) \quad (\theta \leqslant \phi), \tag{1a}$$

$$\tilde{U}_{\mathrm{m}} = 0 \quad (\theta > \phi), \tag{1b}$$

although careful calibrations indicate that the yaw response for $\theta \gtrsim 5°$ has a rather different behaviour. $\tilde{U}_{\mathrm{m}}$ is the measured velocity and ϵ is typically about 0·1 (see Bradbury 1976). Although there is no obvious reason why the deviation from the cosine law should be the same in the (u, v)-plane as it is in the (u, w)-plane, it has been found empirically that ϵ is, in fact, not very sensitive to the velocity vector location in (u, v, w)-space. Whilst the response for $\theta < \phi$ may increase the measurement errors in some circumstances it should be noted that it will in general lead to errors of opposite sign to those arising from the finite yaw response (1 *b*). More importantly, (1 *a*, *b*) still allow fairly straightforward calculations of the expected measurement errors that would occur in practical situations.

For a probe aligned with the plane of its three wires normal to the x-direction ($\psi = 0$, figure 1), and assuming a yaw response given by (1), the mean-velocity component measured by the probe is given by

$$U_{\mathrm{m}} = \iiint_{\mathrm{cone}} u(1+\epsilon v_{\mathrm{r}}/u)\,p(u,v,w)\,du\,dv\,dw, \tag{2}$$

and the mean-square velocity is

$$U_{\mathrm{m}}^2 = \iiint_{\mathrm{cone}} u^2(1+\epsilon\,|v_{\mathrm{r}}|/u)^2\,p(u,v,w)\,du\,dv\,dw, \tag{3}$$

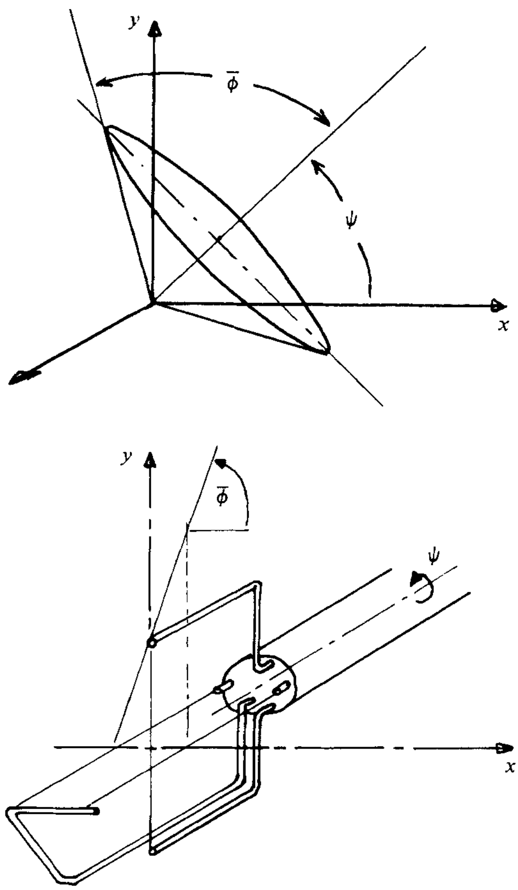

FIGURE 1. Probe geometry and 'acceptance' cone.

where v_r^2 is v^2+w^2, and $p(u,v,w)$ is the joint velocity probability distribution. These expressions assume that the 'missed' heat tracers (those for which arctan $(|v_r|/u) > \bar{\phi})$ are counted as zero velocities rather than being ignored altogether. This was the procedure used in the experiments. The exact form of the probability distribution is not critical for the present purposes, so it is sufficient to assume normality of the three velocity distributions, as was done by Bradbury & Castro (1971) and Bradbury (1976). (Some of the effects of departures from Gaussian distributions are discussed in § 3, when interpreting the experimental data.) Neglecting any correlation between the u- or v- and the w-components of velocity and assuming that the mean lateral component W is zero (i.e. restricting the analysis to the common practical case of two-dimensional mean flow) the joint probability-density distribution is therefore given by

$$p(u,v,w) = A\exp\left[-(u-U)^2/a_1^2-(v-V)^2/a_2^2+2r(u-U)(v-V)/a_{12}^2-w^2/a_3^2\right], \quad (4)$$

where

$$A = [(2\pi)^{\frac{3}{2}}\alpha_u\alpha_v\alpha_w(1-r^2)]^{-1},$$

$$a_1^2 = 2\alpha_u^2(1-r^2), \quad a_2^2 = 2\alpha_v^2(1-r^2), \quad a_3^2 = 2\alpha_w^2, \quad a_{12}^2 = 2\alpha_u\alpha_v(1-r^2);$$

α_u, α_v, α_w are the turbulent intensities $((\overline{u^2})^{\frac{1}{2}}$ etc.) in the x-, y- and z-directions, respectively, and r is the correlation coefficient $\overline{uv}(\overline{u^2}\,\overline{v^2})^{-\frac{1}{2}}$. Some particular, simple cases have been examined previously (Bradbury 1976), but we concentrate here on errors in measurements of the Reynolds stresses. The transverse turbulence energy $\overline{v^2}$ and the shear stress $\overline{uv}$ can be obtained, in an identical way to that sometimes employed using a slant hot wire, by using the geometrical relations

$$\overline{v^2_{\mathrm{m}}} = \overline{u^2_{\mathrm{m}+}} + \overline{u^2_{\mathrm{m}-}} - \overline{u^2_{\mathrm{m}}}, \tag{5a}$$

$$\overline{uv}_{\mathrm{m}} = (\overline{u^2_{\mathrm{m}+}} - \overline{u^2_{\mathrm{m}-}})/2\sin 2\psi - (\overline{u^2_{\mathrm{m}}} - \overline{v^2_{\mathrm{m}}})/2\tan 2\psi, \tag{5b}$$

where suffixes + and − refer to measurements of the mean-square fluctuating energies made with the probe oriented at $+\psi$ and $-\psi$ to the x-direction, and suffix m is used to distinguish measured from exact values. To calculate errors in measurements of $\overline{v^2}$ and $\overline{uv}$ it is obviously first necessary to determine errors in measurements of $\overline{u^2_{\mathrm{m}+}}$ etc; this is conveniently done by applying an appropriate axis transformation to the actual values of $\overline{u^2}$, $\overline{v^2}$ etc. before evaluating the integral in (3) (which invariably then has to be done numerically). Figures 2–5, in which the errors are defined by

$$E_{u^2} = (\overline{u^2_{\mathrm{m}}} - \overline{u^2})/\overline{u^2}, \quad E_{v^2} = (\overline{v^2_{\mathrm{m}}} - \overline{v^2})/v^2, \quad E_{uv} = (\overline{uv}_{\mathrm{m}} - \overline{uv})/\overline{uv},$$

present some results as a function both of the flow variables and the probe yaw-response characteristics.

For a typical probe with $\phi = 70°$ and $\epsilon = 0{\cdot}15$, figure 2 shows the variation in E_{u^2}, E_{v^2} and E_{uv} as the turbulence intensity rises, for a case where the intensities in all three directions are equal, $\sigma_u = \sigma_v = \sigma_w$ (where $\sigma = \alpha/U$) and the correlation coefficient is 0·4, compared with results obtained assuming $\epsilon = 0$, or $\phi = 90°$. To obtain $\overline{uv}$ and $\overline{v^2}$ the probe was supposed to have been rotated through $\pm 45°$, and figure 2(*b*) includes the error in measuring $\overline{v^2}$ with the probe at $\psi = 90°$. The latter results and those for E_{u^2} ($\psi = 0$) are similar to those discussed in earlier work (Bradbury & Castro 1971; Bradbury 1976), but not identical, since here we have taken $\phi = 70°$ and $\epsilon = 0{\cdot}15$ simultaneously and have included a non-zero correlation coefficient. For very low intensities, E_{v^2} with the probe in the transverse ($\psi = 90°$) position tends to -1, independent of ϵ or of ϕ provided that $\phi < 90°$, since at a sufficiently low intensity none of the heat tracers are received by either sensor wire. If $\epsilon = 0$ the error is always negative (Bradbury 1976) but for a sufficiently large ϵ the errors caused by the latter eventually become larger than those caused by the finite yaw response, so that E_{v^2} changes sign – the result in figure 2(*b*) has that behaviour. For $\overline{v^2}$ and $\overline{uv}$ measurements obtained with the probe at $\pm\psi$ it can be shown that, as the turbulent intensity tends to zero, E_{uv} and E_{v^2} tend respectively to $-\epsilon^2$ and $2\epsilon^2$. Figures 2(*b*, *c*) show that the numerical results have that expected behaviour ($\epsilon = 0{\cdot}15$ in this case) and that once the local intensities exceed about 10 % the errors rise very rapidly, principally as a result of the finite yaw response of the probe. Bradbury & Castro pointed out that, when the root-mean-square angle of the fluctuations is close to the yaw limit of the probe, the 'missed' tracers make a strong and erroneous contribution to the intensity measurements, whereas, at higher intensities, the probability of genuine zero-velocity signals occurring is higher and the influence of the missed tracers is then not so significant. This is undoubtedly the reason for the peak errors in $\overline{uv}$ and $\overline{v^2}$ which, for this particular case, occur at turbulent intensities of about 20 % and 30 % respectively.

It is also apparent that the shear-stress error falls subsequently to a minimum before rising again at even higher turbulence levels. Figure 2(*c*) shows that this curious behaviour is also caused by the finite yaw response rather than the imperfect

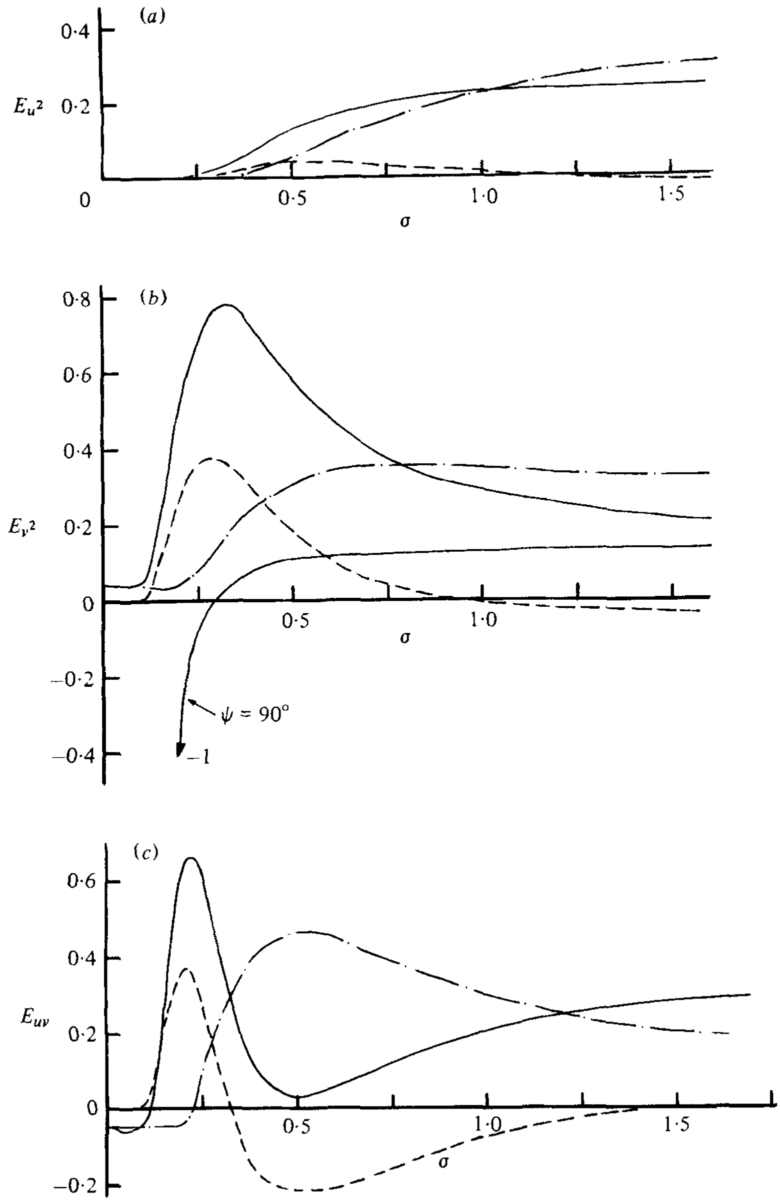

FIGURE 2. PWA errors. $\sigma_u = \sigma_v = \sigma_w$, $V = 0$, $r = 0{\cdot}4$. ——, $\epsilon = 0{\cdot}15$, $\phi = 70°$; – – –, 0, 70°; ·——·, 0·15, 90°. (*a*) E_{u^2}; (*b*) E_{v^2}; (*c*) E_{uv}.

response for $\theta < \phi$ (i.e. $\epsilon \neq 0$). E_{uv} is essentially, of course, the difference in the errors in measurement, at $\pm\psi$, and it is apparent that under some circumstances the latter are of very similar magnitude, so that E_{uv} can be quite small. If there were no correlation between the fluctuating u- and v-components then the problem would be symmetric, so that errors in $\overline{u^2_+}$ and $\overline{u^2_-}$ would be identical, leading to zero error in uv. Otherwise, it is clear that the effect of a non-zero correlation and a finite yaw response is to cause errors in u^2_+ to be larger or smaller than errors in u^2_-, depending on the particular values of r, ϕ and the turbulent intensities.

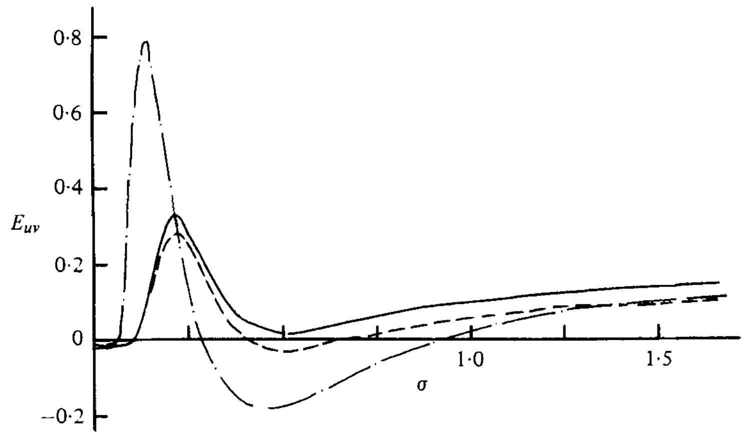

FIGURE 3. Error in shear stress. $\sigma_u = \sigma_v = \sigma_w$, $V = 0$, $r = 0{\cdot}4$. ——, $\epsilon = 0{\cdot}15$, $\phi = 70°$; ·——·, 0·1, 60°; - - -, 0·1, 70°.

Figure 3 shows that the magnitude of the peak errors depends strongly on the yaw limit of the probe; if the geometry is such as to restrict ϕ to 60°, the peak error in $\overline{uv}$ rises to about 80 %, and occurs at a rather lower turbulent intensity. The effects of changes in ϵ are much less significant at the lower intensities, but can be proportionally larger at very high intensities.

In real turbulent flows, the intensities in the three directions are usually far from identical, so it is natural to ask how the errors described above are affected by changes in the relative magnitudes of the three components. Figure 4 (*a*) compares the results shown in figure 2 (E_{uv} and E_{v^2}) with those obtained by taking $\sigma_v^2 = 0{\cdot}5\sigma_u^2$ and $\sigma_w^2 = 0{\cdot}75\sigma_u^2$ – typical values in turbulent shear flows – and figure 4 (*b*) shows how the errors vary with σ_v/σ_u for typical values of σ_u. It is evident that the measurement errors are quite sensitive to the precise characteristics of the turbulence, and it seems probable that accurate determination of errors in a given experiment, where only the measured values are available, is likely to be difficult. This is discussed further in § 3.

A further complication in any practical measurement situation is that the mean lateral velocity component V may be a significant fraction of the longitudinal component. Figure 5 shows that, even for V/U of only 10 %, the errors in $\overline{uv}$ can be significantly different, and even of opposite sign, from those obtained with $V = 0$, whereas the errors in $\overline{v^2}$ are much less dependent on V/U.

Finally, it should be noted that, although, in principle, any value for the probe orientation angle ψ could be used in the determination of $\overline{uv}$ and $\overline{v^2}$, it turns out that $\psi = 45°$ is generally the optimum. We have calculated the errors for a wide range of ψ and, whilst the results are not worth presenting in detail, it was found that the measurement errors were usually smallest for ψ around 45°, which is, perhaps, not a surprising result.

3. Experimental results

3.1. *Techniques*

Crossed-hot-wire and PWA measurements have been made in a nominally 300 mm thick rough-wall boundary layer (§ 3.2), the axisymmetric mixing layer formed downstream of the exit from a 8 in. circular nozzle (§ 3.3), and the shear layer bounding a

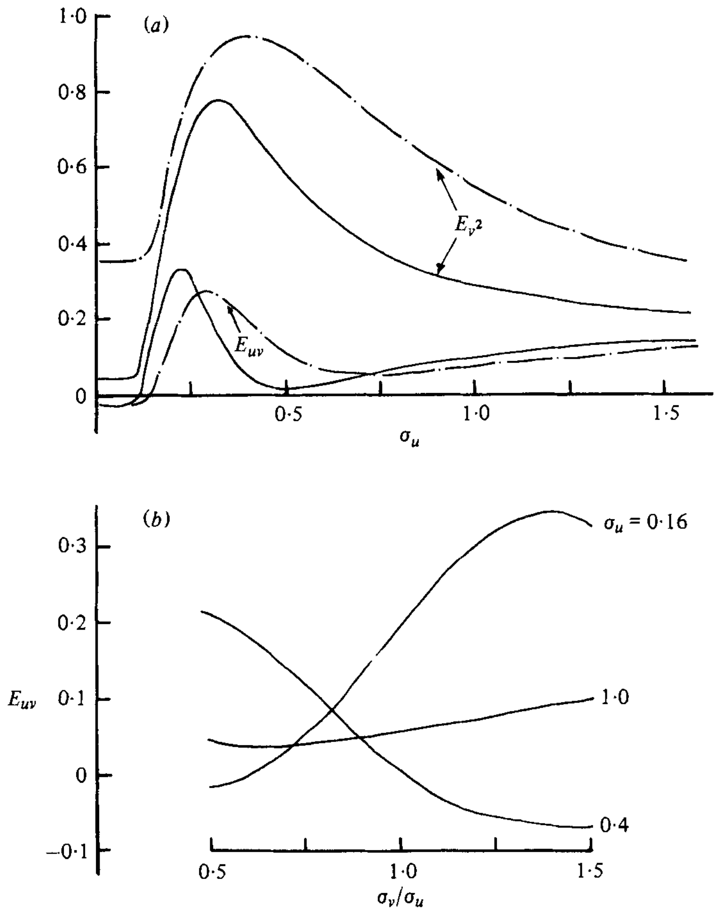

FIGURE 4. The effect of changes in σ_v/σ_u. (*a*) ——, $\sigma_u = \sigma_v = \sigma_w$, $r = 0·4$; ·——·, $\sigma_{v^2} = \frac{1}{2}\sigma_{u^2}$, $\sigma_{w^2} = \frac{3}{4}\sigma_{u^2}$, $r = \overline{uv}/\sigma_u\sigma_u = 0·4$. $\epsilon = 0·15$, $\phi = 70°$. (*b*) $\sigma_w = \sigma_u$, $r = 0·4$.

reversed flow region in the wake of a two-dimensional surface mounted block in the 300 mm thick boundary layer (§ 3.4). The local turbulent intensities therefore ranged from very low to very high, with, in the latter case, mean velocities actually in the reversed direction in some parts of the flow.

All the crossed-wire measurements were made using standard instrumentation. The bridge signals were digitized and analysed on-line using a Commodore PET desk-top computer. Signal linearization was built into the software, much of which was written in machine code, allowing 10000 samples of both channels to be taken in under a minute. Wire calibration, which included a yaw-response calibration using the well-known 'effective cosine-law fit' method (Bradshaw 1971), was also performed on-line. Calibration plus a complete traverse of, say, 20 points, with print-out of all the normalized values of mean velocity and Reynolds stresses, could be achieved within half an hour, so that the effects of hot-wire and electronic drift were reduced to an absolute minimum. Where necessary the cross-wire measurements were corrected for the effects of the lateral (w) component fluctuations and rectification using the results of Tutu & Chevray (1975). Such corrections would have been relatively small in the case of the boundary-layer measurements, since the turbulent intensity only exceeds 20 % in the

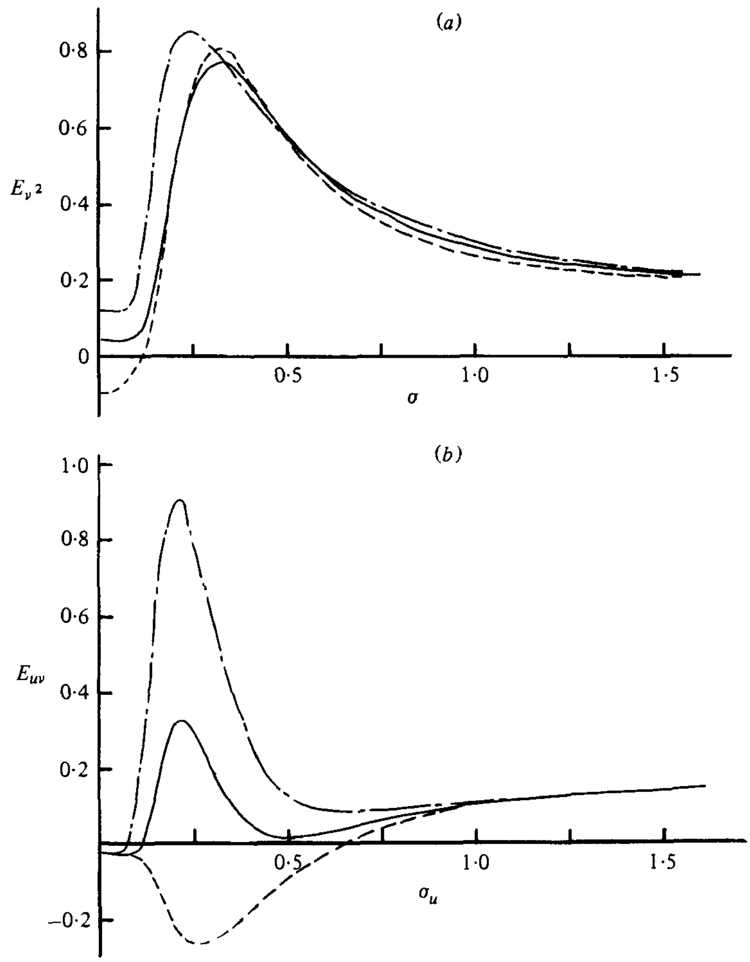

FIGURE 5. The effect of changes in V/U. $\sigma_u = \sigma_v = \sigma_w$, $r = 0{\cdot}4$, $\epsilon = 0{\cdot}15$, $\phi = 70°$. ——, $V/U = 0$; – – –, 0·1; ·——·, −0·1. (a) E_{v^2}; (b) E_{uv}.

bottom 10 % of the layer, but they were significant in the other two flows, where local intensities were much higher. Discussion of the adequacy of these corrections is deferred until later.

The pulsed wire was operated on-line to either a Hewlett-Packard desk-top calculator (for the mixing-layer measurements, § 3.3) or a Hewlett-Packard MX21 mini-computer. In the latter case, the sampling rate, which was limited only by the usual requirement that the pulsed wire should have time to cool before the next 'shot' is fired, was about 50 Hz, allowing 10000 samples to be collected in less than 4 minutes. Using the analysis suggested by Bradbury (1978), it can be shown that the expected *statistical* error in PWA shear-stress measurements for typical cases would be about ± 10 % (with 95 % probability) if 10000 samples were taken for every measurement. In view of the time required to obtain sample sizes necessary to reduce the statistical error significantly, there seemed little point in taking more than 10000 samples; the scatter in the $\overline{uv}$ measurements is, in fact, within ± 10 % in nearly all cases. Rather more scatter was expected (and found) in the case of $\overline{v^2}$ measurements, because each value requires *three* separate measurements ($\psi = \pm 45°$ and $0°$) unless the $\psi = 90°$ probe orientation is used. The probe-rotation mechanisms ensured a relative accuracy in ψ of better than $\frac{1}{4}°$, so errors arising from uncertainties in the exact probe angle were small compared with other sources of error.

3.2. *The boundary-layer measurements*

The thick rough-wall boundary layer was not generated specifically for the investigation, and it is not appropriate here to describe the flow in detail. It was, in fact, developed to simulate the neutrally stable atmospheric boundary layer for studies of bluff-body flows and was generated in the 4 ft 6 in. × 4 ft × 30 ft wind tunnel in the Civil Engineering Department of the University of Surrey, using the well-known Counihan (1969) technique. Crossed-wire and PWA measurements of the mean-velocity and Reynolds stress profiles in the (roughly) fully developed region of the flow (about 9 vorticity-generator heights downstream) are presented in figure 6. The crossed-wire measurements show the expected behaviour (cf. Robins 1979), and the flow is in many ways similar to a naturally grown rough-wall boundary layer. It is clear that, as expected, the PWA data for the mean velocity and longitudinal turbulent energy are quite close to the crossed-wire measurements. If the latter are taken as correct, calculations of the kind described in § 2 suggest PWA mean velocity errors of between 1 and 2 %; clearly the agreement between the two instruments is good.

However, the PWA values for $\overline{uv}$ and $\overline{v^2}$ are invariably significantly higher than the crossed-wire values and, furthermore, are also higher than those expected from the error calculations. The latter are included in figures 6 (*b*, *d*). Only near the wall, where local intensities rise to around 25 %, do the PWA $\overline{v^2}$ measurements agree roughly with the expected values, and here they are about a factor of two higher than the 'actual' values (there is little reason to doubt the crossed-wire measurements in this relatively low-intensity flow). Even at these locations, however, the PWA $\overline{uv}$ measurements are still rather higher than those expected, and elsewhere in the flow both $\overline{uv}$ and $\overline{v^2}$ significantly exceed the expected values. There are only two possible reasons for this. Firstly, at low intensities (less than 10 %, say) the computed errors are quite sensitive to the value of ϵ. Now, whilst yaw-response measurements indicated a value for ϵ of about 0·1 as being appropriate for the whole range of yaw angles, around $\theta = 0°$ *local* values of ϵ would need to be considerably higher than that to give a reasonable fit to the data. In fact, as mentioned earlier, careful yaw calibrations showed that 1 (*a*) is not particularly representative of the response for the very low yaw angles. Secondly, over much of the depth of the boundary layer the flow is intermittent and the mean-velocity probability-density distributions are consequently rather skewed. The assumption of a Gaussian velocity field may not be sufficiently realistic for the purpose of estimating errors in such situations – $\overline{v^2}$, for example, probably depends significantly on the nature of the 'tails' in the probability distributions. In flows with much higher turbulence levels and lower intermittency these features would be less critical, as the results presented in § 3.3 demonstrate. It must be emphasized that the pulsed-wire anemometer need not normally, of course, be used for measurements of the Reynolds stresses in boundary-layer flows, although clearly it is quite capable of making accurate mean-velocity (and longitudinal-intensity) measurements.

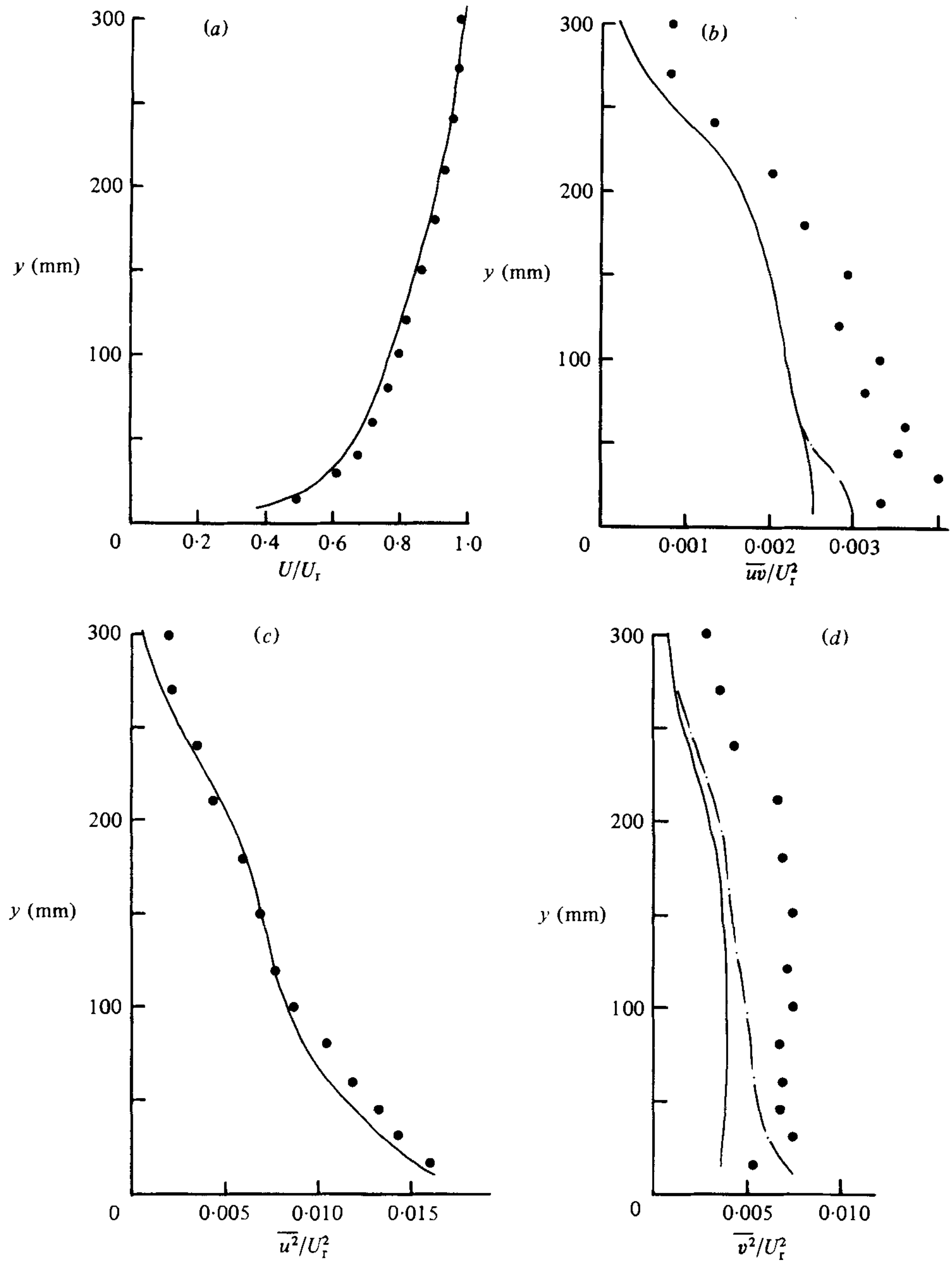

FIGURE 6. 300 mm boundary-layer measurements. ——, crossed-wire data; ·——·, 'expected' pulsed-wire data; ●, actual pulsed-wire data. (*a*) U/U_r; (*b*) $\overline{uv}/U_r^2$; (*c*) $\overline{u^2}/U_r^2$; (*d*) $\overline{v^2}/U_r^2$.

3.3. *The mixing-layer measurements*

Measurements were made through the mixing layer in the initial region of an axisymmetric jet, at about two nozzle diameters downstream from the jet exit, which was preceded by a 11·4:1 contraction. The nozzle diameter was 8 in., the jet exit velocity was typically 8 m/s, and since the nozzle boundary layer was not tripped it was probably laminar at separation. As the intention was simply to make comparative

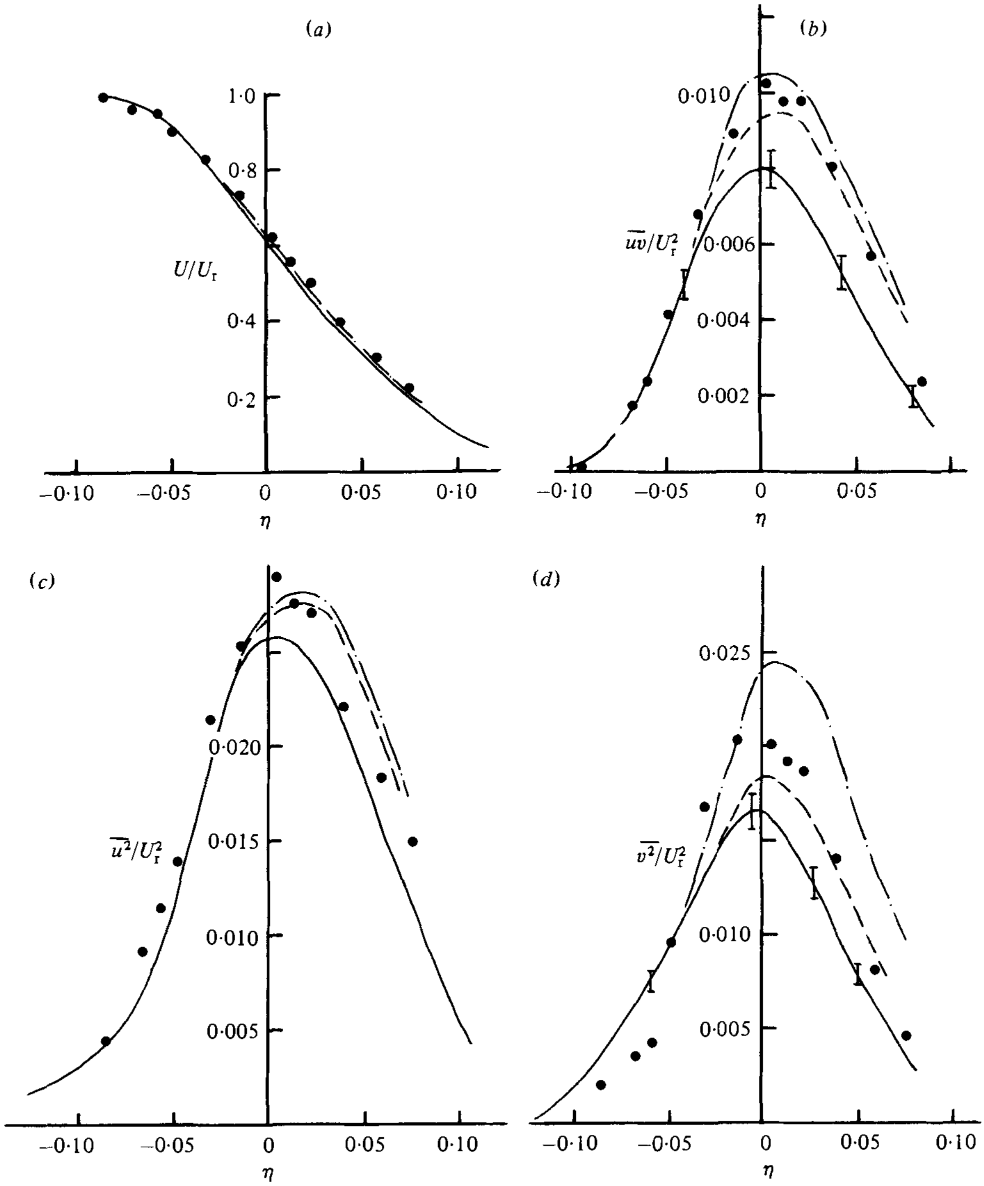

FIGURE 7. Axisymmetric mixing-layer measurements. Legend as in figure 6. - - - -, corrected cross-wire data; I, typical scatter. (*a*) U/U_r; (*b*) $\overline{uv}/U_r^2$; (*c*) $\overline{u^2}/U_r^2$; (*d*) $\overline{v^2}/U_r^2$.

measurements using different instruments, we were not particularly concerned about upstream conditions and the consequent development of the mixing layer, although some additional checks, principally of the growth rate, showed that the flow had characteristics similar to those expected on the basis of classic mixing-layer studies.

Figure 7 shows the variations of mean velocity and Reynolds stresses (except the circumferential component $\overline{w^2}$) measured using a crossed hot wire and a pulsed wire. In the figure η is defined by $\eta = (y-y_0)/(x-x_0)$, where x and y, the axial and radial co-ordinates, respectively, have their origin at the nozzle exit, with y positive outwards from the nozzle edge rather than the symmetry axis, and y_0 and x_0 are the co-ordinates of the mixing layer's virtual origin, deduced by measuring the shear-layer thickness at various downstream locations. Unlike the previous case, local turbulent intensities can be quite large in this flow; even on the centre line ($\eta = 0$) the local axial intensity is about 27 %, and rises rapidly as η increases.

Now Tutu & Chevray (1975) have demonstrated convincingly that crossed hot wires become subject to significant errors once local intensities exceed, say, 20 %. On the basis of their results it is possible to estimate the errors arising in the present crossed-wire measurements, and figure 7 includes the results of correcting the data on that basis. As before, we have taken the liberty of omitting experimental points to avoid confusion; the data exhibited scatter no worse than usual for these types of measurements (typified by the scatter bars included in the figures) and the curves representing the crossed-wire data were obtained simply by drawing 'average' smooth lines through the experimental points. Once local intensities exceed about 50 %, Tutu & Chevray's work suggests that meaningful corrections cannot be made, so the 'corrected' curves do not extend into the low-velocity side of the flow beyond about $\eta = 0{\cdot}07$. Using these corrected results, the expected PWA results were calculated as before and these are included in the figure along with the actual PWA measurements.

The mean-velocity (figure 7*a*) and turbulent-intensity (figure 7*c*) pulsed-wire measurements agree quite well with the values expected from the corrected crossed-wire data, which, in this case also, are not very different from the measured values. The largest corrections to the crossed-wire measurements occur, as expected, in the shear stress and lateral intensity (figures 7*b*, *d*). For the former, the correction is about 20 % near the centre of the mixing layer, but the pulsed-wire error analysis gives an expected error of only about 10 % in addition to this, and the actual pulsed-wire data are encouragingly close to the computed results. Since the pulse-wire calculated error is relatively small, we can conclude that the pulsed-wire and (corrected) crossed-wire measurements are both close to the true shear stress in this flow; published data for $\overline{uv}/U_r^2$ in a mixing layer, and the profile deduced from an integration of the momentum equation, both have a peak value of about 0·01, close to the present results.

On the other hand, the $\overline{v^2}$ pulsed-wire measurements (figure 7*d*) although mostly rather higher than the corrected crossed-wire results, are significantly lower than the expected values. Note that the latter imply an error considerably greater than the $\overline{uv}$ error, as would be anticipated from the results of § 2. Now, since the velocity probability-density distribution in the fully turbulent region around the centre of the layer is quite closely Gaussian (Champagne, Pao & Wygnanski 1976), as assumed in calculation of the pulsed-wire errors, the only possible reasons for the rather lower measurements than those expected are either inappropriate yaw-response assumptions or measurement scatter. Certainly the latter must be more significant than it is in $\overline{uv}$ measurements, as noted earlier. However, we believe that the former is the major cause of the discrepancy, for the following reason. The calculations of the expected errors showed that for positive angles of probe rotation (i.e. in the direction of the shear-stress vector) the probability of flow vectors lying outside the 75° cone (chosen, with $\epsilon = 0{\cdot}1$, as a yaw response fitting the measured response reasonably well) was significantly higher than for rotations in the opposite direction. In fact, the yaw response was, because of the probe geometry, rather better than 75° for negative-angle flow vectors (i.e. $v/u < 0$), and it is precisely those 'effective' flow vectors which contribute most to the probability of missing tracers with the probe at positive orientations (see figure 1). Consequently, a value of ϕ of nearer 80° would have been more appropriate for positive rotations of the particular probe used. This would lead to a significant reduction in the expected pulse wire measurements.

We conclude that the PWA $\overline{v^2}$ measurements were probably no less accurate than the crossed-wire measurements in the high-intensity regions and, in view of the quite large corrections required for the latter may, in fact, be rather more accurate. However, the local intensities near the shear-layer centre line are precisely those which, according to the calculations of § 2, lead to the largest errors in $\overline{v^2}$ measurements, so unless the yaw response of a pulsed wire anemometer were good – extending up to at least 75–80° – one could not in general anticipate particularly accurate measurements. The fact that the probe-mounting arrangement can apparently have a significant effect on the errors at local intensities around 30 %, typically, should be borne in mind when planning experiments of this type.

3.4. *Measurements behind a bluff body*

A nominally 60 mm high block, which was 30 mm thick (in the longitudinal direction) and completely spanned the tunnel working section, was mounted on the surface in the 300 mm rough-wall boundary layer used for the measurements discussed in § 3.2. h/δ, where h is the body height and δ the boundary-layer thickness, was therefore about 0·2, and measurements were made at $x/h = 7{\cdot}0$, with x measured downstream from the front face of the body.

Figure 8 presents the results. Since crossed-wire measurements in regions where the local turbulent intensity exceeds 50 % are very inaccurate and difficult to interpret, no data is included for $y/h \lesssim 1{\cdot}4$, although some single-hot-wire results are shown for interest. Although the forms of all the profiles are very similar to those in the corresponding 'classic' mixing layer (§ 3.3), it must be emphasized that the turbulent intensities are everywhere significantly greater than in the latter case. For example, at the centre of the shear layer (where the Reynolds stresses reach their peak values) the local longitudinal intensity is about 56 %, twice that in the corresponding position in the axisymmetric mixing layer. On the basis of the corrected crossed-wire results, calculations show that the probability of the instantaneous velocity vector lying outside a 45° cone exceeds 15 % at the centre of the shear layer ($y/h \simeq 1{\cdot}5$), so it is clear that even on the higher-velocity side of the layer the crossed-wire results must immediately be treated with caution. We return to this point later.

The results shown in figure 8 have a number of interesting features. Firstly, although the raw cross-wire mean-velocity measurements become increasingly inaccurate as y/h decreases from about 2·0 (where $\sigma_u \simeq 25\%$), the corrected values, the single-hot-wire data, and the pulsed-wire data all agree quite well (the expected pulsed-wire errors are quite small). Below $y/h \simeq 1{\cdot}4$, of course, even the single hot-wire measurements become progressively too high, as expected. Similarly, the corrected crossed-wire longitudinal-intensity measurements agree reasonably well with the single-wire and the pulsed-wire measurements and, again, the expected pulsed-wire errors are relatively small. Since $\overline{v^2}$ is certainly lower than $\overline{u^2}$, we anticipate that even at the very high local intensities around $y/h = 0{\cdot}5$ the errors in $\overline{u^2}$ will be less than those indicated in figure 2(*a*) (for which $\overline{u^2} = \overline{v^2}$), and therefore probably nowhere exceed about 15 %.

Secondly, however, and in direct contrast to the corresponding results presented in § 3.3, the pulsed-wire $\overline{uv}$ and $\overline{v^2}$ data lie consistently and significantly above the

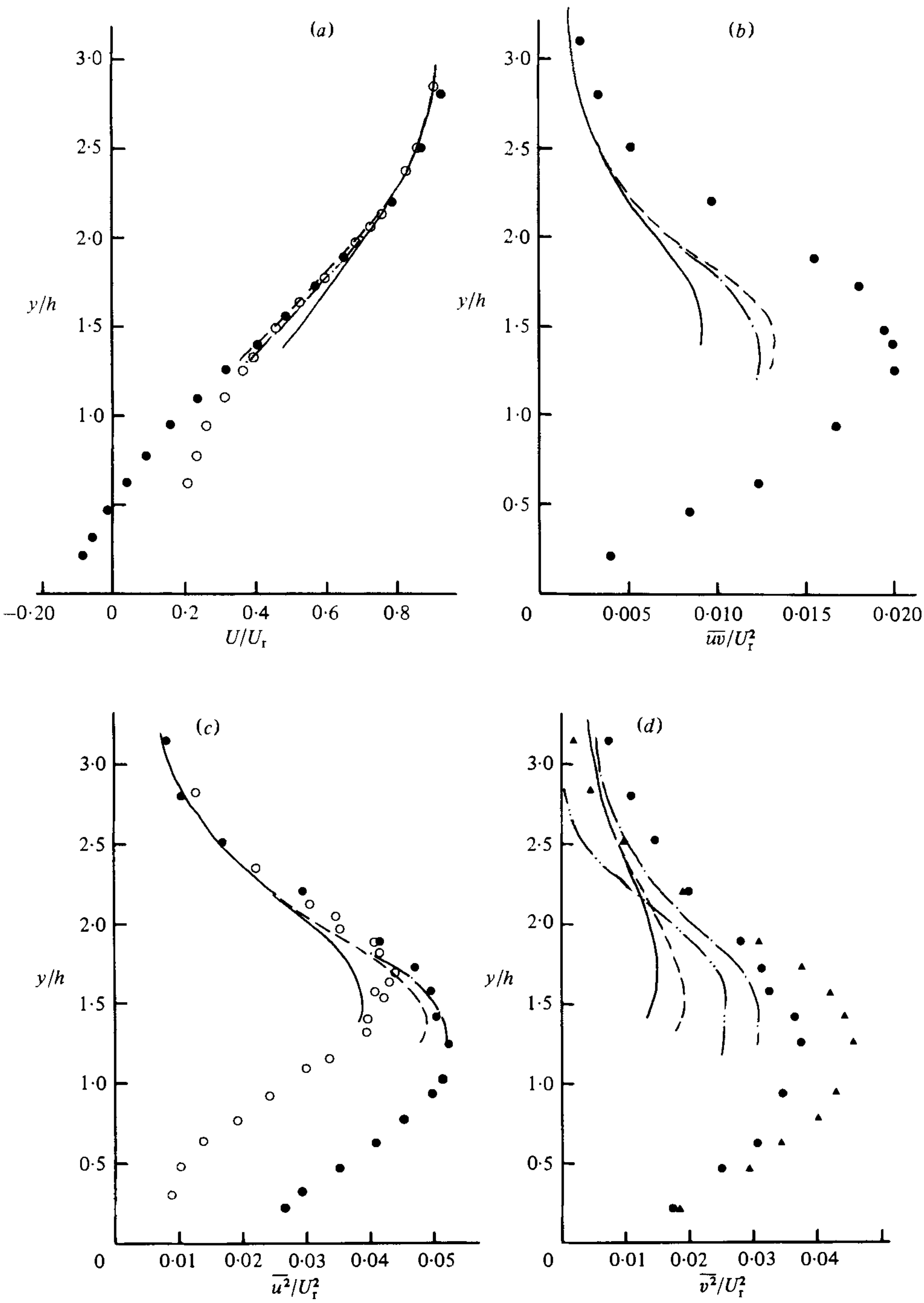

FIGURE 8. Mixing layer behind two-dimensional block in 300 mm boundary layer. ($x/h = 7$, $\delta/h \simeq 5$). Legend as in figure 6. (*a*) U/U_r, ○, single hot wire; (*b*) $\overline{uv}/U_r^2$; (*c*) $\overline{u^2}/U_r^2$; (*d*) $\overline{v^2}/U_r^2$. ▲, pulsed wire for $\psi = 90°$; —‥—, 'expected' PWA data for $\psi = 90°$.

expected values, even where local intensities are similar to those at the centre of the axisymmetric mixing layer. Now both the raw crossed-wire and the pulsed-wire data indicated a significant mean V-component throughout the flow ($V/U = -0{\cdot}05$ at $y/h = 3{\cdot}0$, rising to $-0{\cdot}15$ at $y/h = 1{\cdot}5$). We have demonstrated earlier that this has a significant effect on errors in pulsed-wire measurements, particularly of $\overline{uv}$ (figure 5*b*). It seems probable, as Tutu & Chevray actually suggest, that crossed-wire errors must also depend significantly on the magnitude of the V-component, and consequently the corrected crossed-wire results shown in figures 8 (*b*, *d*), used to estimate the expected pulsed-wire measurements, may well be seriously underestimated, because they were based on the assumption that $V = 0$ – Tutu & Chevray did not extend their analysis to cases where $V \neq 0$. Since, on the basis of the results shown in § 2, we would anticipate rather *lower* pulsed-wire errors than those that occur in the lower-intensity axisymmetric mixing layer (comparison of the results in figure 7 and 8 confirms this), this seems the only reasonable explanation for the larger discrepancy between the crossed-wire and the pulsed-wire data. A recent survey of measurements in the shear layer bounding the reversed flow region behind a rearward-facing step (Bearman, private communication) confirms our conclusion that crossed hot wires tend to underestimate the Reynolds stresses in such flows, even when corrections are applied, because of the significantly higher intensities and transverse mean velocities that occur, compared to those in a plane mixing layer.

The final point concerns the pulsed-wire measurements of $\overline{v^2}$ made with the probe in the $-90°$ orientation. At the high-velocity edge of the flow, where the intensities are relatively low, these $\overline{v^2}$ values are lower than both the crossed-wire data and the results obtained with the probe in the $\pm 45°$ and $0°$ positions, as anticipated (see figure 2*b*). However, in the central region the results are higher, in direct contrast to the implications of figure 2 (*b*) and the actual expectations for this flow, calculated using the corrected crossed-wire data. This is almost certainly a result of non-Gaussianity of the velocity distributions; the probe at $\psi = 90°$ will measure velocity vectors representing the 'tails' of the probability distributions and miss those around the mean-velocity direction (i.e. those near the centre of the distribution). In this special case, therefore, the calculated pulse-wire errors will be particularly sensitive to the form of the distribution, and figure 2 (*b*) ($\psi = 90°$) might consequently not give a very good indication of the errors occurring in real flows.

It is encouraging, however, that the pulsed-wire data for $\overline{v^2}$ obtained in both ways agrees to within about 20 % around the centre of the shear layer and nearer the wall, where the intensities are very high, the agreement is even better, as expected.

4. Conclusions

In general terms, the measurements described in § 3 confirm the overall theoretical deductions (§ 2) about the form of the errors to be expected in pulsed-wire measurements of the Reynolds stresses, except in flows of low intensity (less than, say, 10 %). In this latter case, the form of the velocity probability-density distribution and, more particularly perhaps, the exact nature of the yaw response, are important in determining the errors; it seems that, whilst measurements of mean velocity and longitudinal turbulent intensity can be quite accurate (as shown in previous work), measurements of lateral intensity and shear stress are usually not. This is of limited practical importance since ordinary hot-wire techniques are, of course, quite adequate in low-intensity flows.

More significant is the conclusion that, in flows of such high intensity that hot wires would be useless, measurements of all the Reynolds stresses can be made with an accuracy probably better than 30 % (for $\overline{v^2}$) or even 15 % (for $\overline{u^2}$ and $\overline{uv}$). In the medium-intensity range (10–30 %, say) it has been shown that, provided the yaw response extends to large enough angles, pulsed-wire measurements can be as accurate as hot-wire measurements. However, in this case the errors in $\overline{uv}$ and $\overline{v^2}$ are rather greater than at higher intensities and depend critically on the extent of the yaw response and, consequently, on those probe-geometrical asymmetries that affect that response differently in different flow-angle 'quadrants'.

In view of the complicated way in which the various errors arise and their non-monotonic variation with changes in the flow parameters, it is extremely unlikely that any *general* procedure could be devised to allow correction of the measured stresses. However, if the yaw response of the probe were known (it should always be measured if Reynolds-stress measurements are to be attempted) and the local turbulent intensities were clearly higher than those at which errors are likely to be a maximum (for $\overline{v^2}$) or a minimum (for $\overline{uv}$), then figures 2–5 could certainly be used to estimate the magnitude of the errors. It appears from both the theoretical and the experimental results that the errors can be reduced to quite acceptable levels, provided only that the probe's yaw response extends up to at least 80°. In making Reynolds-stress measurements it will therefore often be worth accepting a reduction in the maximum flow velocity at which the probe can be used by mounting the sensor wires rather closer to the pulsed wire than is currently usual, simply in order to maximize the yaw-response range. Final confirmation of the quantitative accuracy of pulsed-wire anemometer measurements of Reynolds stresses in highly turbulent flows must clearly await comparative measurements made with a different instrument of inherently higher accuracy. Laser anemometry is certainly the only other technique capable of making measurements in highly turbulent flows, but there is still considerable debate concerning its accuracy when used to measure Reynolds stress (in air).

In conclusion, we believe that it is possible to make useful measurements of the Reynolds stresses in highly turbulent flows using a pulsed-wire anemometer. Whilst such measurements can never be highly accurate, they need be no more inaccurate than are cross-wire measurements in lower-intensity flows and would therefore be quite adequate for many purposes.

The authors wish to acknowledge useful discussion with Dr L. J. S. Bradbury and the technical expertise of Mr T. Laws, without which the experiments would have been impossible. Thanks are also due to Dr N. Toy for help in various aspects of interfacing the pulsed-wire anemometer with the Hewlett-Packard MX21 mini-computer, and to Mr R. Northam for help in running some of the experiments.

REFERENCES

BRADBURY, L. J. S. 1969 *N.P.L. Aero Rep.* no. 1284.

BRADBURY, L. J. S. 1976 *J. Fluid Mech.* **77**, 473.

BRADBURY, L. J. S. 1978 Examples of the use of the pulsed wire anemometer in highly turbulent flow. In *Dynamic Measurements in Unsteady Flow*, p. 489. Proc. Dynamic Flow Conf. 1978, P.O. Box 121, Skovlunde, Denmark.

BRADBURY, L. J. S. & CASTRO, I. P. 1971 *J. Fluid Mech.* **49**, 657.

BRADSHAW, P. 1971 *An Introduction to Turbulence and its Measurement.* Pergamon.

BRITTER, R. E. & HUNT, J. C. R. 1979 *J. Ind. Aero.* **4**, 165.

CASTRO, I. P. 1971 *J. Fluid Mech.* **46**, 599.

CASTRO, I. P. & ROBINS, A. G. 1977 *J. Fluid Mech.* **79**, 307.

CASTRO, I. P. & SNYDER, W. H. 1982 *Atmos. Environ.* (to appear).

CHAMPAGNE, F. H., PAO, Y. H. & WYGNANSKI, I. J. 1976 *J. Fluid Mech.* **74**, 209.

COUNIHAN, J. 1969 *Atmos. Environ.* **3**, 197.

EATON, J. K., JOHNSTON, J. P. & JEANS, A. H. 1979 In *Proc. 2nd Int. Symp. on Turbulent Shear Flows, London, Paper* no. 16.7.

ROBINS, A. G. 1979 *J. Ind. Aero.* **4**, 71.

TUTU, N. K. & CHEVRAY, R. 1975 *J. Fluid Mech.* **71**, 785.